Endogenous Depth of Reasoning and Response Time, with an application to the Attention-Allocation Task

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Abstract

We extend Alaoui and Penta's (2015, 2016a,b) model of Endogenous Depth of Reasoning to account for Response Time, and apply it to generate predictions for Avoyan and Schotter's (2015) attention allocation task. We show that all the experimental findings of Avoyan and Schotter are consistent with this model.

1 Preamble

Alaoui and Penta (2015, 2016a,b) introduce a model in which the number of *steps of reasoning* a player performs results from a cost-benefit analysis:[1] each step of reasoning entails a certain understanding of the strategic situation, and is associated to an incremental cost and value; the agent continues reasoning as long as the incremental value is larger than the cost. While it seems tempting to identify depth of reasoning with response time, the model of Alaoui and Penta does not refer directly to the latter. The reason is that the unit of measure of a step of reasoning is not time, but a 'unit of understanding'. But the amount of time needed to achieve a certain 'unit of understanding' may vary from case to case: if it is harder to reason about G^1 than G^2, then any given 'unit of understanding' may take longer in G^1, and hence even if the two games induced the same depth of reasoning (i.e., level of understanding), they may induce different response times.

In this short note we extend Alaoui and Penta's (2015, 2016a,b) model of Endogenous Depth of Reasoning to account for response time, and apply it to generate predictions for Avoyan and Schotter's (2015, AS hereafter) *attention allocation task*. We show that this model, appended with one simple assumption on the cognitive equivalence classes,

[1] Alaoui and Penta (2015) provides an axiomatic foundation to this model, by characterizing the properties of the (unobservable) reasoning process which justify a cost-benefit approach, as well as particular functional forms for the value of reasoning. Alaoui and Penta (2016a) applied this model to level-k reasoning in games, provide an empirical test, and showed how the model can be used to perform robust predictions across games as well as explain the experimental findings in Goeree and Holt (2001). Alaoui and Penta (2016b) generalizes the model to general (non level-k) models of reasoning in coordination games, and show how heterogeneity of cognitive abilities between interacting agents favor equilibrium coordination in games of initial response, provided that players agree on their relative sophistication.

delivers predictions that are confirmed by the experimental findings in AS (in fact, all of AS's findings are consistent with this model). Thus, not only the model provides a simple theory to explain AS's interesting findings, but it shows that AS's experimental results provide strong support to the approach put forward in Alaoui and Penta (2015, 2016a,b).

Section 2 provides an informal explanation of the model and results. Section 3 contains the formal model and the application to the attention allocation task.

2 Informal Explanation

2.1 The Baseline EDR Model

Alaoui and Penta (2015, 2016a,b) introduce a model in which the number of *steps of reasoning* a player performs results from a cost-benefit analysis: each step of reasoning entails a certain understanding of the strategic situation, and is associated to an incremental cost and value; the agent continues reasoning as long as the incremental value is larger than the cost. The cost of reasoning represents cognitive abilities of the agent, and is pinned down by the *cognitive equivalence class* of the game, which groups together games that are 'essentially equivalent', in the sense that they have the same strategic structure and they are equally difficult to reason about (cf. Section 2.2 in Alaoui and Penta (2015)). The value of reasoning instead is related to the payoffs of the game, and such that – at a minimum – it increases as the payoff differences are increased. Thus, if two games G^1 and G^2 belong to the same equivalence class, and G^1 has higher payoff differences than G^2, then a simple comparative statics exercise implies that the depth of reasoning is higher in G^1 than G^2: the cost of reasoning is the same, whereas the value of reasoning is higher in G^1. If, on the other hand, the two games are *not* cognitively equivalent, then in principle both cost and value of reasoning may be shifting at the same time, and hence the comparative statics is indeterminate in general.

The three key concepts in this model therefore are the *cost of reasoning*, the *value of reasoning*, and the *cognitive equivalence partition*. Alaoui and Penta (2015) take the latter as given, and provide foundations to alternative functional forms for the value of reasoning and use them to perform comparative statics exercise *within* a cognitive equivalence class.[2] Comparisons *across* equivalence classes and the investigations of the determinants of the costs of reasoning are left to future research. However, it is clear that *if* two games, G^1 and G^2, have the same value of reasoning, but G^1 has a higher cost of reasoning than G^2, then they belong to distinct cognitive equivalence classes and the depth of reasoning should be lower in G^1.

[2] Alaoui and Penta (2016a) also accounts for players' beliefs and higher order beliefs over the opponents' costs. We abstract from these issues here because they do not affect the results below.

2.2 EDR and Response Time

As mentioned above, the model of Alaoui and Penta does not directly refer to response time, and the unit of measure of a step of reasoning is not time but a 'unit of understanding'. Since the amount of time needed to achieve a certain 'unit of understanding' may vary from case to case, even if two games induce the same depth of reasoning (i.e., level of understanding), they may induce different response times.

Based on this logic, one should thus expect that if two games are cognitively equivalent, and as such equally difficult to think about, then every step of reasoning would take an equal amount of time in the two games. Hence, if G^1 induces a higher depth of reasoning than G^2, and they are cognitively equivalent, then we expect a higher response time in G^1 than G^2. Combining this insight with the determinants of the value of reasoning from Alaoui and Penta (2015), we can say the following: *If G^1 and G^2 are cognitively equivalent, and G^1 has higher payoff differences than G^2, then we expect a higher response time in G^1 than in G^2.*

Comparisons of response times across cognitive equivalence classes, however, incur the following problem: if G^1 and G^2 have the same value of reasoning, but G^1 has a higher cost of reasoning than G^2, then the number of steps performed would be lower in G^1, but each of them would take longer than in G^2. Overall, the resulting effect on response time is indeterminate. If, however, we could ensure that the depth of reasoning (not the value) is the same in G^1 and G^2 (which could be achieved if G^1 had both higher cost and value of reasoning), then the response time would be higher in G^1. Thus: *If G^1 and G^2 are not cognitively equivalent, and if G^1 is harder to reason about than G^2, then: (i) if the value of reasoning is the same in the two games, then the effect on response times is indeterminate; (ii) if the depth of reasoning is the same in the two games, then we expect a higher response time in G^1.*

2.3 The Attention Allocation Task

Avoyan and Schotter's (2015, AS hereafter) start from the following observation: Individuals in the real world are involved in several games at the same time, and if they are cognitively bounded, they must choose the attention they allocate to different games. Then, they play according to the chosen attention allocation. But since behavior in one game arguably depends on the attention allocated to it, and the attention allocated to a game G^1 arguably depends on which other game the individual is engaged in, it follows that understanding an individuals' behavior in G^1 necessarily involves considering the entire context (e.g., whether it's $\left(G^1, G^2\right)$ or $\left(G^1, \hat{G}^2\right)$), and how the attention is allocated over competing tasks.

To shed light on this question, AS conduct an experiment in which subjects are presented a sequence of pairs of games $\left(G^1, G^2\right)$. For each pair, they are asked the fraction $\alpha^l \in [0, 1]$ of a total time X to allocate to game G^l, where $l \in \{1, 2\}$ and $\alpha^1 + \alpha^2 = 1$.

Later in the experiment, one such pair would be drawn at random, and the subject asked to play games G^1 and G^2, each of which with the previously time constraints: that is, time $\alpha^l X$ for G^l.

AS provide a rich set of data in which a variety of games are compared. These games include the Prisoner's Dilemma (PD), Battle of the Sexes (BoS), Constant Sum (CS) and Pure Coordination (PC), each with different specification of the payoffs, and a 'Chance' game, which is strategically equivalent to a BoS game. The main findings are the following: (i) for any class of games (PD, BS, PC, CS, Chance), 'increasing the payoff differences' in the game increases the attention allocated to that game; (ii) if zero entries in the CS games are substituted by non-zero entries, but without changing the baseline strategic structure, then the attention allocated to that game increases; (iii) on average, the highest attention is allocated to PD, followed by CS, then BoS and PC, although the latter two induce very similar shares of allocated attention.

Given the observations in Section 2.2, the EDR-model can be extended to derive testable predictions for AS's attention allocation task. In this task, subjects are presented a pair of games $\left(G^1, G^2\right)$, and they are asked the fraction $\alpha^l \in [0,1]$ of a total time X to allocate to game G^l ($\alpha^1 + \alpha^2 = 1$). In the following we explain how all the experimental results in AS are consistent with an extension of Alaoui and Penta's (2015, 2016a) model of Endogenous Depth of Reasoning (EDR), appended with one simple assumption on the cognitive equivalence classes, which formalizes one of AS's working hypothesis.

In particular, consider a two-stage procedure in which first the agent is presented a pair $\left(G^1, G^2\right)$, and chooses the fraction $\alpha^l \in [0,1]$ to allocate to game G^l. Then, in the second stage, the agent plays the game G^l according to Alaoui and Penta's model, with induced response time $T\left(G^l\right)$. (The time constraint would be binding if $\alpha^l X < T\left(G^l\right)$). Clearly, in the first stage the agent chooses the time allocation without having really reasoned about the two games. Hence, he would not know how much time he will actually need or whether a given time allocation will be binding or not, nor would he have a precise understanding of how he will play in the game. Assume, however, that the agent's choice in the first stage is 'consistent' with his future reasoning process in the following (weak) sense:

> **(A.1)** for any pair $\left(G^1, G^2\right)$, α^1 is increasing in $T\left(G^1\right)$, decreasing in $T\left(G^2\right)$, and such that $\alpha^1 > \alpha^2$ if $T\left(G^1\right) > T\left(G^2\right)$.

Concerning the equivalence classes in the baseline EDR model, assume that:

> **(C.1)** whenever two games induce the same pure-action best responses *and* have the same zeros, they are cognitively equivalent; and **(C.2)** if a game is changed turning some zeros into non-zeros, but without changing the strategic structure, then the game becomes 'harder to reason about'.

Note that (C.1) is implied by Alaoui and Penta's (2015) conditions (Section 2.2), whereas (C.2) merely formalizes one of Avoyan and Schotter's (2015) working hypothesis in terms of a restriction on Alaoui and Penta's (2015) cognitive equivalence classes. (A.1) is a mild assumption, which enables applying Alaoui and Penta's model to the Attention Allocation Task.

Under these assumptions, Alaoui and Penta's model generates the following predictions in AS's attention allocation task, all of which are consistent with AS's findings:[3]

1. the time-allocation relation is *transitive*.

2. it predicts all the findings on 'monotonicity within game classes' (Table 7 of AS) and on the Equity Hypothesis (Table 8 of AS), at a statistically significant level.

3. The model's predictions on the pairwise comparisons in AS's C-group, which we summarize in Section 3.3 (page 10).

4. The pairwise comparisons for games outside of AS's C-group (cf. Table 5 in AS, summarized here in Section 3.3, page 10).

3 Model

We focus on static games with complete information, $G = (A_i, u_i)_{i \in I}$, where I is the set of players, for each $i \in I$, A_i is the (finite) set of actions of player i and letting $A := \times_{i \in I} A_i$, $u_i : A \to \mathbb{R}$ is player i's payoff function. For simplicity, we assume that G is such for every $a_{-i} \in A_{-i} := \times_{j \neq i} A_j$ and for every $i \neq j$, player i has a single best-response, which we denote $BR_i(a_{-i})$.

3.1 Baseline EDR-Model

In situations of strategic uncertainty, such as games of initial response, players' beliefs over their opponents' behavior may only stem from their own individual reasoning, and have no guarantee of being correct. We thus model individuals' reasoning processes in isolation from each other. In particular, we assume that each player $i \in I$ forms conjectures about the opponents' behavior as he reasons further. Such a *path of reasoning* is described by a sequence of profiles $\{(a_i^{i,k}, a_{-i}^{i,k})\}_{k \in \mathbb{N}_+}$ where $a_{-i}^{i,k}$ denotes i's k-th step *conjecture* about the opponents behavior, and $a_i^{i,k}$ his own k-th step action. This notion of path of reasoning encompasses different kinds of reasoning processes. In particular, the model includes as special cases level-k reasoning (s.t. $a_i^{i,k} = BR_i\left(a_{-i}^k\right)$), deliberation on equilibrium selection (in which $a_j^{i,k} = BR_j(a_{-j}^k)$ for all j's), etc., which we don't discuss in the interest of brevity.

Besides allowing general reasoning processes, we depart from standard models of limited reasoning (e.g., Nagel (1995), Costa-Gomes and Crawford (1995), Camerer, Ho and Chong (2004), etc.) in that a player's depth of reasoning arises endogenously in our model, and specifically through a cost-benefit analysis, for which we provide foundations in Alaoui

[3]We stress that these are *all* the predictions of the model for this experiment. Hence, *not one* single finding in AS contradicts the theoretical predictions.

and Penta (2015). In particular, we assume that the cost of reasoning $c_i : \mathbb{N}_+ \to \mathbb{R}_+$ is such that $c_i(1) = 0$, and that c_i is increasing and unbounded for each i, and that the value of reasoning $v_i : \mathbb{N}_+ \to \mathbb{R}_+$ is such that

$$v_i(k+1) = \max_{a_{-i} \in A_{-i}} u_i(BR_i(a_{-i}), a_{-i}) - u_i(a_i^{i,k}, a_{-i}). \tag{1}$$

This functional form, which we provide an axiomatic foundation for in Alaoui and Penta (2015, Theorem 5), corresponds to a maximum gain representation of v_i. Intuitively, it is as if the value attached to taking the $k+1$-th step of reasoning corresponds to the highest payoff improvement i could obtain, relative to his current action $a_i^{i,k}$. We use this functional only for simplicity here, as the model's implications we discuss below hold for a much more general class of value of reasoning functions (e.g., the representations in Theorems 1, 2 and 3 in Alaoui and Penta (2015)).

Given the cost and value of reasoning, player i keeps reasoning as long as the value exceeds the cost. Formally, for any $(c, v) \in \mathbb{R}_+^{\mathbb{N}_+} \times \mathbb{R}_+^{\mathbb{N}_+}$, let

$$\mathcal{K}(c, v) = \min\{k \in \mathbb{N}_+ : c(k) \leq v(k) \text{ and } c(k+1) > v(k+1)\}. \tag{2}$$

Player i's depth of reasoning is then determined by the value that this function takes at (c_i, v_i), and denoted

$$\hat{k}_i = \mathcal{K}(c_i, v_i). \tag{3}$$

Then, player i chooses the action according to his depth of reasoning,[4] that is $a_i^* = a_i^{i,\hat{k}_i}$.

The Cognitive Equivalence Classes: The advantage of a cost-benefit approach is that it allows for comparative statics exercises on the depth of reasoning. But for these to be meaningful, there must be a way of shifting the value without changing the cost or the path of reasoning itself. For instance, we would not compare an easier game with low stakes to a harder one with high stakes and conclude that, due to the higher incentives, the agent attains a deeper understanding in the latter. That is because the two problems would also differ in the *cost of reasoning*. We therefore introduce a notion of 'cognitive equivalence class', which groups together problems that are equally difficult and approached essentially in the same way. The representation theorems in Alaoui and Penta (2015) imply that the costs are uniquely pinned down within each class. This ensures that cognitively equivalent problems only differ in the value of reasoning they entail, thereby allowing for meaningful comparative statics on depth of reasoning, within each cognitive

[4] In a more general version of the model (cf. Alaoui and Penta, 2016a,b), a player's depth of reasoning need not pin down his behavior, which may also depend on his beliefs over the opponents. In that case, the depth of reasoning represents a player's *capacity*, which varies systematically with the payoffs of the game (through the dependence of v_i on the game's payoffs). This capacity is binding if player i believes he is playing an opponent who has reached a deeper understanding than he has, but it is not otherwise (see Alaoui and Penta (2016a) for an experimental test of the model's predictions when beliefs are varied). Here we abstract from these details, which would not affect the implications for attention allocation that we discuss below.

equivalence class.

Consider the following definitions:

Definition 1 *Two games are* path-equivalent *if they induce the same path of reasoning; they are* cost-equivalent *if they are associated to the same cost-of-reasoning; they are* cognitively equivalent *if they are both path- and cost-equivalent.*

Definition 2 *Take two path-equivalent games, G^1 and G^2, with associated costs of reasoning c_i^1 and c_i^2. We say that G^1 is* 'harder to reason about' *than G^2 (for player i) if and only if $c_i^1(k) > c_i^2(k)$ for each $k > 1$.*

We maintain the following assumptions on the cognitive equivalence classes:[5]

Assumption 1 *Take any pair (G^1, G^2):*

1. *If G^1 and G^2 induce the same pure-action best-response functions $BR_i : A_{-i} \rightarrow A_i$ for all players, then they are path-equivalent. If they also have the same zeros, then they are cost-equivalent.*

2. *If G^2 is obtained from G^1 by replacing the zeros with non-zero entries, without changing the strategic structure, then G^2 is 'harder to reason about' than G^1.*

For example, consider the following games:

G^1	L_2	R_2		G^2	L_2	R_2		G^3	L_2	R_2
T_1	$3,1$	$0,0$		T_1	$6,2$	$0,0$		T_1	$3,1$	$0,1$
B_1	$0,0$	$1,3$		B_1	$0,0$	$3,6$		B_1	$1,0$	$1,3$

According to Assumption 1, games G^1 and G^2 are cognitively equivalent, and hence they are associated with the same path and costs of reasoning. Moreover, the payoff differences are higher in G^2, and hence it entail a higher value of reasoning than G^1 (see, e.g., eq. 1). It follows that G_1 induces a higher depth of reasoning than G^2.

In contrast, G^3 is not cognitively equivalent to G^1, because some zero entries were turned into non-zeros, leaving everything else unchanged. Then, by Assumption 1.2, G^3 has a higher cost of reasoning than G^1. In this case, it also has a lower value of reasoning, so it will induce a lower depth of reasoning. As will be discussed next, however, the effect on response time is ambiguous in this case.

[5] Part 1 is the same assumption made in Alaoui and Penta (2016), and it is implied by the maintained assumption on cognitive equivalence classes in Alaoui and Penta (2015). Part 2 instead formalizes, within the language of Alaoui and Penta (2015), one of the hypothesis stated in Avoyan and Schotter (2015).

3.2 Extension to Response Time

For any given game G, let $\tau_i : \mathbb{N}_+ \to \mathbb{R}_+$ denote the *time function*, where $\tau_i(k)$ represents the time it takes to perform the k-th step of reasoning. Then, the *total response time* in game G, given the associated cost, value and time functions (c_i, v_i, τ_i) is equal to:

$$T_i(G) \equiv T_i(c_i, v_i, \tau_i) := \sum_{k=1}^{\mathcal{K}(c_i,v_i)} \tau_i(k).$$

As discussed in the introduction, it seems reasonable to assume that if it's harder to attain a certain understanding in one game than in another, then not only is the cost of reasoning (weakly) higher, but it also takes (weakly) more time to attain that understanding. We thus connect the baseline model of Alaoui and Penta (2016a,b) to response times, by means of the following simple assumption:

Assumption 2 *Take any two path-equivalent games, G^1 and G^2, with associated costs of reasoning c_i^1 and c_i^2, and time functions τ_i^1 and τ_i^2. Then, for every $k \in \mathbb{N}_+$: $c_i^1(k) \geq c_i^2(k)$ implies $\tau_i^1(k) \geq \tau_i^2(k)$.*

Remark 1 *If G^1 and G^2 are cognitively equivalent, but G^2 has larger payoff differences, then $T_i(G^2) \geq T_i(G^1)$. But if the two games are not cognitively equivalent, then the comparison between $T_i(G^2)$ and $T_i(G^1)$ is indeterminate.*

3.3 Application: The Attention Allocation Task

As described in the previous section, in Avoyan and Schotter's (2015) experiment subjects are presented a sequence of pairs of games (G^1, G^2). For each pair, they are asked the fractions $\alpha^1, \alpha^2 \in [0,1]$ s.t. $\alpha^1 + \alpha^2 = 1$ of a total time X to allocate to each of the two games. Later in the experiment, one such pair would be drawn at random, and the agent asked to play games G^1 and G^2, each of which with the previously time constraints: that is, for each $l = 1, 2$, time $\alpha^l X$ for G^l.

We let $\alpha_{AB}(A)$ denote the fraction allocated to game A in the pairwise comparison (A, B). AS provide the following notions of consistency for these choices:

Property 1 (Transitivity) *If $\alpha_{AB}(A) > \alpha_{AB}(B)$ and $\alpha_{BC}(B) > \alpha_{BC}(C)$, then $\alpha_{AC}(A) > \alpha_{AC}(C)$.*

Property 2 (Baseline Independence) *If $\alpha_{AX}(A) > \alpha_{BX}(B)$ for some game X, then $\alpha_{AZ}(A) > \alpha_{BZ}(B)$ for all games Z.*

Property 3 (Baseline Consistency) *If $\alpha_{AX}(A) > \alpha_{BX}(B)$ for some game X, then $\alpha_{AB}(A) > \alpha_{AB}(B)$.*

Property 4 (IIA) *If $\alpha_{AB}(A) > \alpha_{AB}(B)$, then $\alpha_{ABC}(A) > \alpha_{ABC}(B)$.*

The model above can be easily adapted to this 'attention allocation task', by positing a simple two-stage procedure in which first the agent is presented a pair $\left(G^1, G^2\right)$, and chooses $\alpha \in [0,1]$. Then, in the second stage, he applies the model of endogenous depth of reasoning above to game G^1 (given the cost, value and time functions it induces), but subject to the time constraint, so that the resulting depth of reasoning is

$$\hat{k}_i^* = \min\left\{\mathcal{K}\left(c_i, v_i\right), \kappa\left(\alpha\right)\right\},$$
$$\text{where } \kappa\left(\alpha\right) := \max\left\{k : \sum_{l=1}^{k} \tau_i\left(l\right) \leq \alpha X\right\}.$$

(that is, the actual depth of reasoning is the same as in the baseline model if the time constraint αX is not binding, otherwise it coincides with the highest k reached within the time limit).

Clearly, in the first stage, the agent chooses the time allocation without really reasoning over the two games, and hence without necessarily having a precise understanding of how much time he will need in the two games. However, as long as the agent is minimally forward-looking, a reasonable assumption is that his time allocation choices are aligned with his actual (future) reasoning, in the sense that if $T_i\left(G^1\right) > T_i\left(G^2\right)$, then in a direct comparison between G^1 and G^2, the agent anticipates that he'll need more time in G^1 than G^2, and his time allocation choice reflects this. The following assumption formalizes such a minimal notion of consistency:

Assumption 3 *For any attention allocation task* $\left(G^1, G^2\right)$*,* α^1 *is increasing in* $T_i\left(G^1\right)$*, decreasing in* $T_i\left(G^2\right)$*, and* $\alpha^1 > \alpha^2$ *if* $T_i\left(G_1\right) > T_i\left(G_2\right)$.

Remark 2 It can be shown that, under Assumptions 1-3, Alaoui and Penta's model has the following predictions for the attention-allocation task*, all of which are confirmed by Avoyan and Schotter's experiment (cf. Tables 1,2,5,7,8 in AS):*[6]

1. The patterns of choice satisfy *transitivity*, but not necessarily the other consistency properties.

2. The predictions for the pairwise comparisons in AS's C-group are summarized in the

[6] We stress that these are *all* the predictions of the model for this experiment (in particular, the model offers no predictions for the comparisons in Tables 3,4, and 6). Hence, *not one* finding in AS contradicts the model's predictions.

following table (cf. Tables 1-2 in AS):[7]

	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*
(1) $Chance$	=	=	<	=	<						
(2) PC_{500}	=	=	<	=	<						
(3) PC_{800}	>	>	=	>	=						
(4) BS_{500}	=	=	<	=	<						
(5) BS_{800}	>	>	=	>	=						
(6) CS_{500}						=	<				
(7) CS_{800}						>	=				
(8) CS_{400}								=			
(9) PD_{800}									=	>	>
(10) PD_{500}									<	=	>
(11) PD_{300}									<	<	=

(The model's predictions in the table above are in terms of weak ordering, and they are all confirmed by AS data. In fact, with the exception of the $(BS_{800}, Chance)$ and (PC_{800}, BS_{500}) comparisons, for which AS find that the time allocation is no different from uniform at a statistically significant level, all the other data are consistent with the theoretical predictions in the strict ordering sense, at a statistically significant level.)

3. For the pairwise comparisons for games outside of AS's C-group (cf. Table 5 in AS):[8]

	PC_{800}	$PC_{\frac{800}{500}}$	$PC_{\frac{800}{500}100}$
PC_{500}	<	<	*
PC_{800}		>	*
$PC_{\frac{800}{500}}$			*

	BS_{500}	$BS_{\frac{800}{0}}$	$BS_{\frac{800}{100}}$
BS_{800}	>	*	*
BS_{500}		*	<
$BS_{\frac{800}{0}}$			*

	PD_{500}	PD_{300}	$PD_{\frac{800}{500}}$
PD_{800}	>	>	*
PD_{500}		>	*
PD_{300}			*

	CS_{500}	CS_{400}
CS_{500}	<	*
CS_{800}		*

(All these data are consistent with the theoretical predictions at a statistically significant level.)

4. The predictions for the pairwise comparisons in AS's Tables 7 (on 'monotonicity

[7] A '>' entry (respectively, '<' and '=') means that the time allocated to the game in the corresponding row is predicted to be weakly higher than (respecitvely, 'weakly lower than' and 'equal to') that in the corresponding column. Other cells are left blank because the model offers no predictions there, as those are comparison across cognitive equivalence classes.

[8] The convention is the same as above, except that now the entry '∗' indicates that the model's predictions are indeterminate, as the the games belong to different cognitive equivalence classes. The other entries are left blank because they are also blank in the Table 5 in AS.

within game classes') and 8 (the Equity Hypothesis) are the same as in AS's data (all statistically significant).

Remark 3 We note that, since the individual is significantly time constrained in the first stage of the attention allocation task, he compares games G^1 and G^2 based on a very superficial assessment, compared to the criteria he may later adopt to reason about the two games in the second stage. This suggests that the relevant cognitive equivalence partition in the attention allocation stage need not coincide with that in the actual choice task: games that appear cognitively equivalent based on a fast, superficial examination, may actually induce different paths of reasoning, or different costs, once the reasoning actually unfolds. By this logic, one may thus expect the relevant c.e. partition for the first stage to be coarser than that for the second stage. For instance, Assumption 1.1 could be strengthened by requiring that two games are path equivalent for player i at stage-1, whenever they induce the same BR_i for player i (as opposed to *for all* players (cf. Assumption 1.1), which is maintained for stage-2). This extension would enrich the set of predictions for the attention allocation task, for instance by putting the CS-games in the same c.e. class as the PC, BS and Chance games. In the above, for the sake of simplicity, we ignored the conceptually important distinction between c.e. partition in the first and second stage. We note, however, that even with this first-stage coarsening of the c.e. partition, all the extra predictions of the model for the attention allocation task are consistent with AS's experimental findings.

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